9-2 Reflections

Common Core State Standards

G-CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure . . . .
Also G-CO.A.2, G-CO.A.4, G-CO.B.6

MP 1, MP 3, MP 4

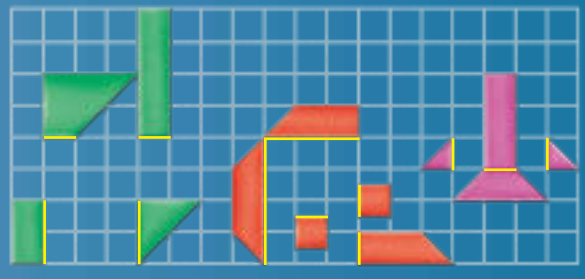
Objective To find reflection images of figures

If it's hard to visualize, try using grid paper to draw the shapes.

MATHEMATICAL PRACTICES

Getting Ready!

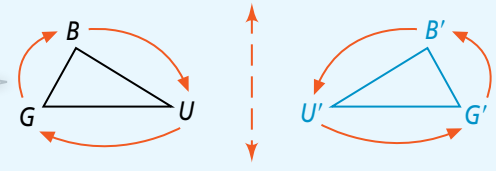
Look at the shapes at the right. Visualize flipping each shape across its yellow line. What word do the images of the shapes form? Copy the shapes as they are shown and sketch the results of flipping them.



In the Solve It, you reflected shapes across lines. Notice that when you reflect a figure, the shapes have *opposite orientations*. Two figures have opposite orientations if the corresponding vertices of the preimage and image read in opposite directions.

Lesson Vocabulary
- reflection
- line of reflection

The vertices of $\triangle BUG$ read clockwise.



The vertices of $\triangle B'U'G'$ read counterclockwise.

Essential Understanding When you reflect a figure across a line, each point of the figure maps to another point the same distance from the line but on the other side. The orientation of the figure reverses.

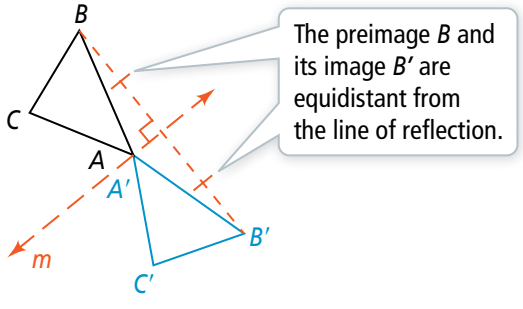
take note

Key Concept Reflection Across a Line

A **reflection** across a line m, called the **line of reflection,** is a transformation with the following properties:

- If a point A is on line m, then the image of A is itself (that is, $A' = A$).
- If a point B is not on line m, then m is the perpendicular bisector of $\overline{BB'}$.

You write the reflecion across m that takes P to P' as $R_m(P) = P'$.



The preimage B and its image B' are equidistant from the line of reflection.

You can use the equation of a line of reflection in the function notation. For example, $R_{y=x}$ describes the reflection across the line $y = x$.

Problem 1 Reflecting a Point Across a Line

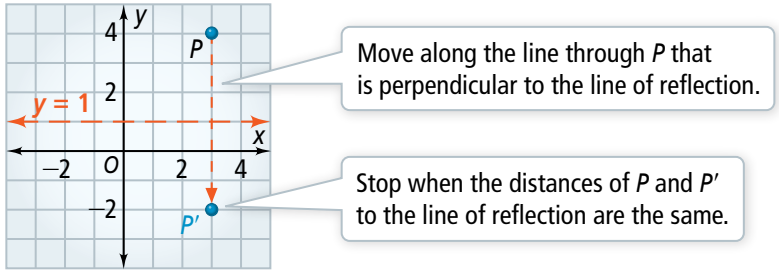
Multiple Choice **Point P has coordinates (3, 4). What are the coordinates of $R_{y=1}(P)$?**

(A) $(3, -4)$ (B) $(0, 4)$ (C) $(3, -2)$ (D) $(-3, -2)$

Graph point P and the line of reflection $y = 1$. P and its reflection image across the line must be equidistant from the line of reflection.

Think

How does a graph help you visualize the problem?
A graph shows that $y = 1$ is a horizontal line, so the line through P that is perpendicular to the line of reflection is a vertical line.



P is 3 units above the line $y = 1$, so P' must be 3 units below the line $y = 1$. The line $y = 1$ is the perpendicular bisector of $\overline{PP'}$ if P' is $(3, -2)$. The correct answer is C.

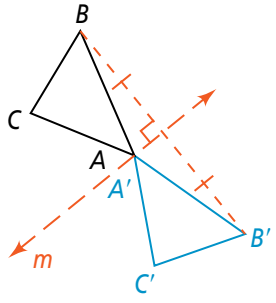
Got It? **1.** $R_{x=1}(P) = P'$. What are the coordinates of P'?

You can also use the notation R_m to describe reflections of figures. The diagram below shows $R_m(\triangle ABC)$, and function notation is used to describe some of the properties of reflections.

take note

Property Properties of Reflections

- Reflections preserve distance.
  If $R_m(A) = A'$, and $R_m(B) = B'$, then $AB = A'B'$.
- Reflections preserve angle measure.
  If $R_m(\angle ABC) = \angle A'B'C'$, then $m\angle ABC = m\angle A'B'C'$.
- Reflections map each point of the preimage to one and only one corresponding point of its image.
  $R_m(A) = A'$ if and only if $R_m(A') = A$.



Observe that the above properties mean that reflections are rigid motions, which you learned about in Lesson 9-1.

Problem 2 Graphing a Reflection Image

Coordinate Geometry **Graph points $A(-3, 4)$, $B(0, 1)$, and $C(4, 2)$. Graph and label $R_{y\text{-axis}}(\triangle ABC)$.**

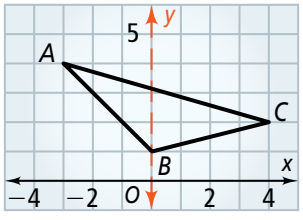
Step 1
Graph $\triangle ABC$. Show the y-axis as the dashed line of reflection.

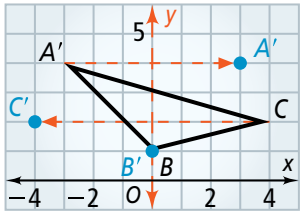
Step 2
Find A', B', and C'. B' is in the same position as B because B is on the line of reflection. Locate A' and C' so that the y-axis is the perpendicular bisector of $\overline{AA'}$ and $\overline{CC'}$.

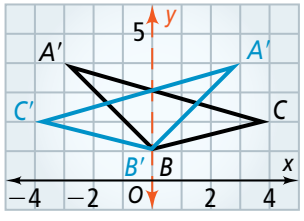
Step 3
Draw $\triangle A'B'C'$.

Think

$\triangle ABC$ intersects the line of reflection. How will the image relate to the line of reflection?
The image will also intersect the line of reflection.







Got It? **2.** Graph $\triangle ABC$ from Problem 2. Graph and label $R_{x\text{-axis}}(\triangle ABC)$.

Problem 3 Writing a Reflection Rule

Each triangle in the diagram is a reflection of another triangle across one of the given lines. How can you describe Triangle 2 by using a reflection rule?

Plan

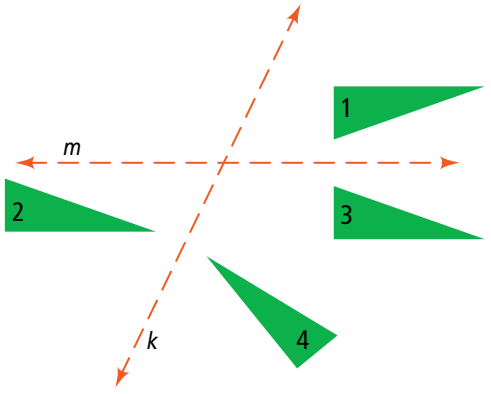
If Triangle 2 is the image of a reflection, what do you know about the preimage?
The preimage has opposite orientation, and lies on the opposite side of the line of reflection.

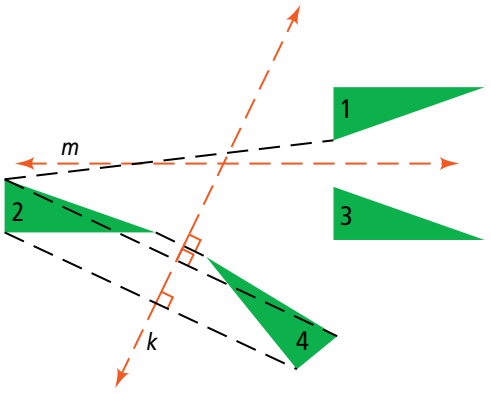
Triangle 2 is the image of a reflection, so find the preimage and the line of reflection to write a rule.

The preimage cannot be Triangle 3 because Triangle 2 and Triangle 3 have the same orientation and reflections reverse orientation.

Check Triangles 1 and 4 by drawing line segments that connect the corresponding vertices of Triangle 2. Because neither line k nor line m is the perpendicular bisector of the segment drawn from Triangle 1 to Triangle 2, Triangle 1 is not the preimage.

Line k is the perpendicular bisector of the segments joining corresponding vertices of Triangle 2 and Triangle 4. So, Triangle 2 = R_k(Triangle 4).





Got It? **3.** How can you use a reflection rule to describe Triangle 1? Explain.

You can use the properties of reflections to prove statements about figures.

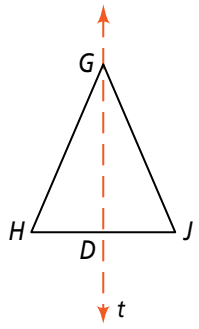
Problem 4 Using Properties of Reflections

Plan

What do you have to know about $\triangle GHJ$ to show that it is an isosceles triangle?
Isosceles triangles have at least two congruent sides.

In the diagram, $R_t(G) = G$, $R_t(H) = J$, and $R_t(D) = D$. Use the properties of reflections to describe how you know that $\triangle GHJ$ is an isosceles triangle.

Since $R_t(G) = G$, $R_t(H) = J$, and reflections preserve distance, $R_t(\overline{GH}) = \overline{GJ}$. So, $GH = GJ$ and, by definition, $\triangle GHJ$ is an isosceles triangle.



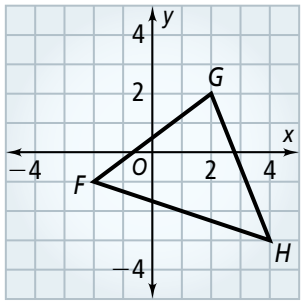
Got It? **4.** Can you use properties of reflections to prove that $\triangle GHJ$ is equilateral? Explain.

Lesson Check

Do you know HOW?

Use the graph of $\triangle FGH$.

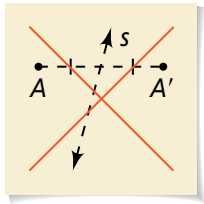
1. What are the coordinates of $R_{y\text{-axis}}(H)$?
2. What are the coordinates of $R_{x=3}(G)$?
3. Graph and label $R_{y=4}(\triangle FGH)$.



Do you UNDERSTAND? MATHEMATICAL PRACTICES

4. **Vocabulary** What is the relationship between a line of reflection and a segment joining corresponding points of the preimage and image?

5. **Error Analysis** A classmate sketched $R_s(A) = A'$ as shown in the diagram.



   a. Explain your classmate's error.

   b. Copy point A and line s and show the correct location of A'.

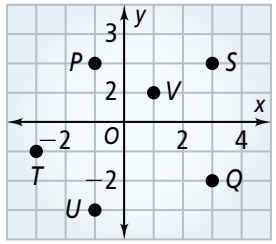
6. What are the coordinates of a point $P(x, y)$ reflected across the y-axis? Across the x-axis? Use reflection notation to write your answer.

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find the coordinates of each image.

7. $R_{x=1}(Q)$
8. $R_{y=-1}(P)$
9. $R_{y\text{-axis}}(S)$
10. $R_{y=0.5}(T)$
11. $R_{x=-3}(U)$
12. $R_{x\text{-axis}}(V)$



See Problem 1.

Coordinate Geometry Given points $J(1, 4)$, $A(3, 5)$, and $G(2, 1)$, graph $\triangle JAG$ and its reflection image as indicated. See Problem 2.

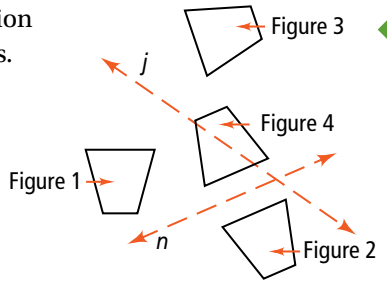
13. $R_{x\text{-axis}}$ **14.** $R_{y\text{-axis}}$ **15.** $R_{y=2}$ **16.** $R_{y=5}$ **17.** $R_{x=-1}$ **18.** $R_{x=2}$

19. Each figure in the diagram at the right is a reflection of another figure across one of the reflection lines. See Problem 3.

a. Write a reflection rule to describe Figure 3. Justify your answer.

b. Write a reflection rule to describe Figure 2. Justify your answer.

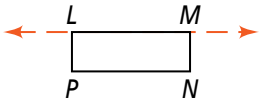
c. Write a reflection rule to describe Figure 4. Justify your answer.



20. In the diagram at the right, $LMNP$ is a rectangle with $LM = 2MN$. See Problem 4.

a. Copy the diagram. Then sketch $R_{\overline{LM}}(LMNP)$.

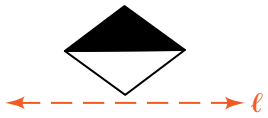
b. What figure results from the reflection? Use properties of reflections to justify your solution.



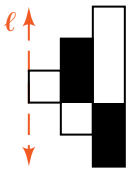
B Apply

Copy each figure and line ℓ. Draw each figure's reflection image across line ℓ.

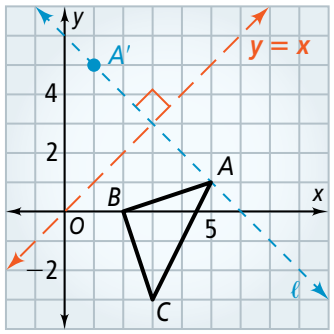
21.



22.



23. **Coordinate Geometry** The following steps explain how to reflect point A across the line $y = x$.



Step 1 Draw line ℓ through $A(5, 1)$ perpendicular to the line $y = x$. The slope of $y = x$ is 1, so the slope of line ℓ is $1 \cdot (-1)$, or -1.

Step 2 From A, move two units left and two units up to $y = x$. Then move two more units left and two more units up to find the location of A' on line ℓ. The coordinates of A' are $(1, 5)$.

a. Copy the diagram. Then draw the lines through B and C that are perpendicular to the line $y = x$. What is the slope of each line?

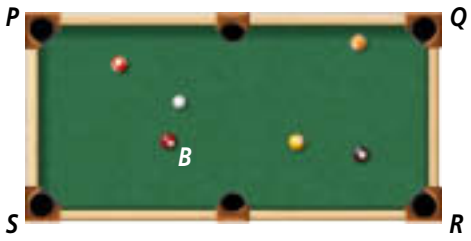
b. $R_{y=x}(B) = B'$ and $R_{y=x}(C) = C'$. What are the coordinates of B' and C'?

c. Graph $\triangle A'B'C'$.

d. **Make a Conjecture** Compare the coordinates of the vertices of $\triangle ABC$ and $\triangle A'B'C'$. Make a conjecture about the coordinates of the point $P(a, b)$ reflected across the line $y = x$.

24. **Coordinate Geometry** $\triangle ABC$ has vertices $A(-3, 5)$, $B(-2, -1)$, and $C(0, 3)$. Graph $R_{y=-x}(\triangle ABC)$ and label it. (*Hint:* See Exercise 23.)

25. Recreation When you play pool, you can use the fact that the ball bounces off the side of the pool table at the same angle at which it hits the side. Suppose you want to put the ball at point B into the pocket at point P by bouncing it off side $\overline{RS}$. Off what point on $\overline{RS}$ should the ball bounce? Draw a diagram and explain your reasoning.



26. Think About a Plan The coordinates of the vertices of $\triangle FGH$ are $F(2, -1)$, $G(-2, -2)$, and $H(-4, 3)$. Graph $\triangle FGH$ and $R_{y=x-3}(\triangle FGH)$.

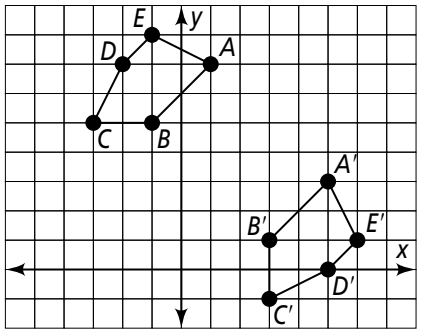
- What is the relationship between the line $y = x - 3$ and $\overline{FF'}$, $\overline{GG'}$, and $\overline{HH'}$?
- How can you use slope to find the image of each vertex?

27. In the diagram $R(ABCDE) = A'B'C'D'E'$.

a. What are the midpoints of $\overline{AA'}$ and $\overline{DD'}$?

b. What is the equation of the line of reflection?

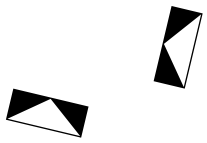
c. Write a rule that describes this reflection.



Copy each pair of figures. Then draw the line of reflection you can use to map one figure onto the other.

28.

29.



30. History The work of artist and scientist Leonardo da Vinci (1452–1519) has an unusual characteristic. His handwriting is a mirror image of normal handwriting.

a. Write the mirror image of the sentence, "Leonardo da Vinci was left-handed." Use a mirror to check how well you did.

b. Explain why the fact about da Vinci in part (a) might have made mirror writing seem natural to him.

31. Open-Ended Give three examples from everyday life of objects or situations that show or use reflections.

Find the image of $O(0, 0)$ after two reflections, first across line ℓ_1 and then across line ℓ_2.

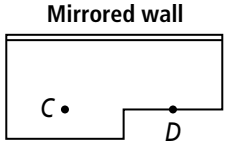
32. $\ell_1: y = 3$, ℓ_2: x-axis

33. $\ell_1: x = -2$, ℓ_2: y-axis

34. ℓ_1: x-axis, ℓ_2: y-axis

35. Reasoning When you reflect a figure across a line, does every point on the preimage move the same distance? Explain.

36. Security Recall that when a ray of light hits a mirror, it bounces off the mirror at the same angle at which it hits the mirror. You are installing a security camera. At what point on the mirrored wall should you aim the camera at C in order to view the door at D? Draw a diagram and explain your reasoning.



37. Use the diagram at the right. Find the coordinates of each image point.

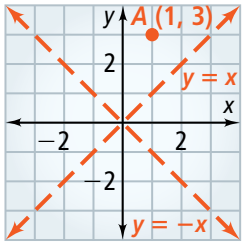
a. $R_{y=x}(A) = A'$

b. $R_{y=-x}(A') = A''$

c. $R_{y=x}(A'') = A'''$

d. $R_{y=-x}(A''') = A''''$

e. How are A and A'''' related?



Challenge

Reasoning **Can you form the given type of quadrilateral by drawing a triangle and then reflecting one or more times? Explain.**

38. parallelogram

39. isosceles trapezoid

40. kite

41. rhombus

42. rectangle

43. square

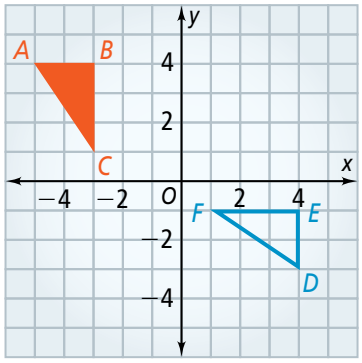
44. Coordinate Geometry Show that $R_{y=x}(A) = B$ for point $A(a, b)$ and $B(b, a)$. (*Hint:* Show that $y = x$ is the perpendicular bisector of $\overline{AB}$.)

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 1

Look back at the information about the video game on page 543, and review the requirements for Case 1 and Case 2 of the video game program. The graph of the puzzle piece and target area is shown again below.

Select all of the following that are true. Explain your reasoning.



A. $T_{<2,\ -2>}(x, y)$ moves $\triangle ABC$ so that the image of C is the reflection of F across the y-axis.

B. $T_{<2,\ -2>}(x, y)$ moves $\triangle ABC$ so that the image of $\triangle ABC$ is the reflection of $\triangle DEF$ across the y-axis.

C. $R_{y=x}(C) = F$

D. F is the reflection of $(-1, 1)$ across the line $y = x$.

E. F is the reflection of C across the line $y = 2x + 2$.

F. E is the reflection of B across the line $y = 2x + 2$.

G. The image of the puzzle piece after a reflection across a line has the same orientation as the target area.

H. The problem cannot be solved for Case 1 using only reflections.